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TESTS OF THIN HEMISPHERICAL SHELLS SUBJECTED TO INTERNAL HYDROSTATIC PRESSURE

A REPORT OF TESTS

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

IN COÖPERATION WITH

THE CHICAGO BRIDGE AND IRON COMPANY

BY

WILBUR M. WILSON

AND

JOSEPH MARIN

BULLETIN No. 295
ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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TESTS OF THIN HEMISPHERICAL SHELLS SUBJECTED TO INTERNAL HYDROSTATIC PRESSURE

I. Introduction

1. *Object of Tests.*—Large spherical shells fabricated of steel plates are used as vessels for storing gases under pressure. The walls of these vessels are subjected to a tension having the same intensity in all directions in a plane. The question has been raised in connection with the design of these vessels whether or not a plate subjected to stress in all directions can resist the same intensity of stress as a similar plate subjected to stress in one direction only. This is a question which has been discussed by many writers, and which has been the basis for a large amount of experimentation. The tests reported in this bulletin simulate very closely the conditions of service in a large spherical storage vessel containing gas under pressure, and it is believed that the results are directly applicable to the design of such vessels.

The specimen used in these tests was a thin hemispherical shell connected to a thick base by means of a short cylindrical section continuous with, and having the same radius as, the hemisphere. Tests were made on shells of two materials, steel of structural grade and an aluminum alloy.

2. *Acknowledgments.*—These tests are a part of the research work of the Engineering Experiment Station of the University of Illinois, of which Dean M. L. Enger is the director, and of the Department of Civil Engineering, of which Prof. W. C. Huntington is the head. The steel specimens were contributed by the Lukens Steel Company and the aluminum specimens by the Aluminum Company of America. The steel shells were tested by Joseph Marin, a research graduate assistant in Civil Engineering. A thesis containing a report of these tests was submitted by Mr. Marin in partial fulfillment of the requirements for the degree of master of science in Civil Engineering in the Graduate School of the University of Illinois, 1930. Information relative to these tests contained in this bulletin was taken from Mr. Marin's thesis, which was written under the supervision of Prof. Wilbur M. Wilson. The aluminum shells were tested by Frederic B. Metterhausen, working under the supervision of Prof. Wilson. The funds to meet the direct expenses of the investigation were contributed by the Chicago Bridge and Iron Company.

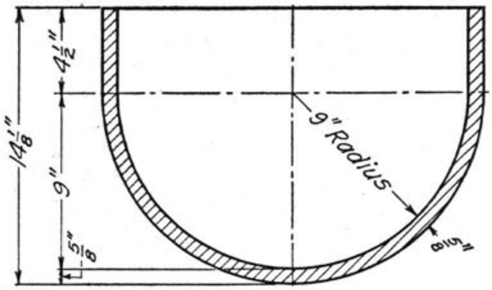

FIG. 1. UNFINISHED STEEL SPECIMEN

TABLE 1
PROPERTIES OF MATERIAL OF SPECIMENS 30-1 AND 30-2

Chemical Analysis				Physical Properties		
Carbon	Manganese	Phosphorus	Sulphur	Yield Point lb. per sq. in.	Ultimate Tensile Strength lb. per sq. in.	Elongation per cent
0.220	0.380	0.010	0.034	36 900	57 600	33

II. STRENGTH OF HEMISPHERICAL SHELLS

3. *Tests of Steel Shells.*—The two steel specimens, designated as 30-1 and 30-2, were machined from shells spun to the dimensions shown in Fig. 1. The chemical and physical properties of the material, as reported from mill tests, are given in Table 1. Preliminary to the tests, the unfinished specimens were stress-relieved by holding them at a temperature of 1575 deg. F. for one hour and then allowing them to cool in the furnace. They were then machined to the dimensions shown in Fig. 2. It is to be noted that the spherical portion, which constitutes the specimen proper, was machined carefully both inside and out, and that the transition from the thin to the thick portion was gradual so as to eliminate any sharp reentrant angles. The outer end of the cylindrical portion was threaded so as to screw into the thick base, as shown in Fig. 3. After the specimen had been machined, the thickness was measured at regular intervals over the entire surface, and the results of the measurements at the various points are given in Fig. 4.

The apparatus for producing the internal pressure is shown in Fig. 5. The weighted plunger C served as an accumulator and also

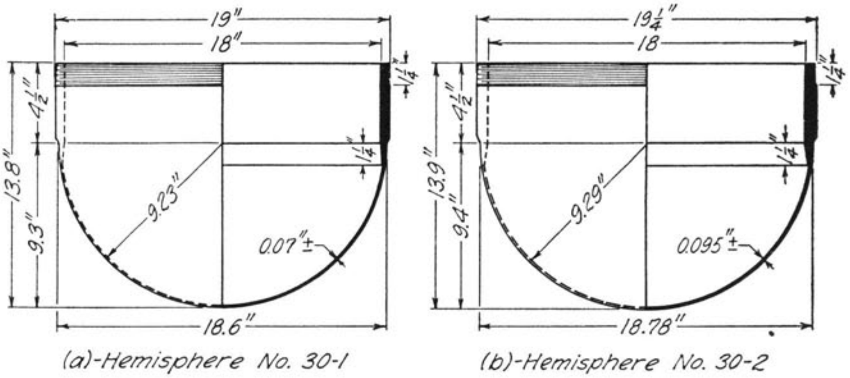

FIG. 2. FINISHED SPECIMENS 30-1 AND 30-2

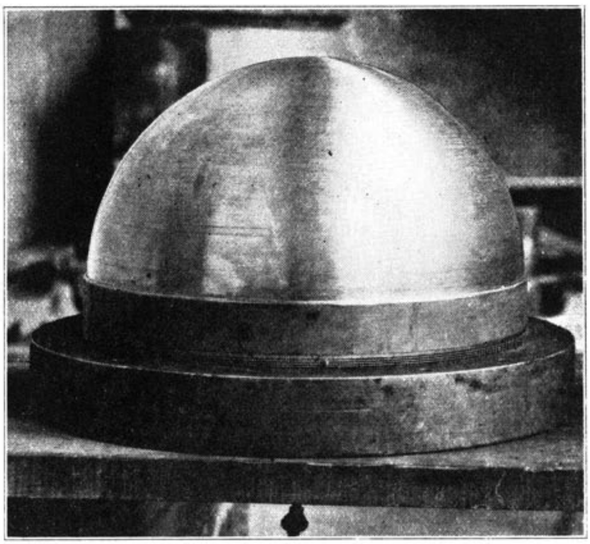

FIG. 3. SPECIMEN 30-1 SCREWED INTO BASE READY TO BE TESTED

as a check upon the hydraulic gage D. In making a test, enough weights were placed upon C to produce the desired intensity of pressure within the spherical shell. The plunger carrying the weights was rotated so as to eliminate friction as a source of error.

The strain in the shell was measured by means of two optical strain gages, one mounted at the top, and the other at one side at an angle of 45 degrees, as shown in Fig. 5. The strain gage consisted of a micrometer microscope mounted in such a manner that it could be

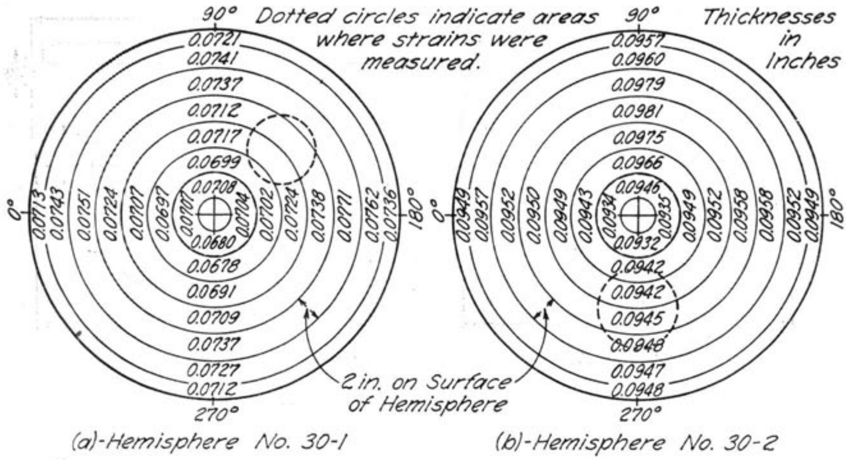

FIG. 4. THICKNESS OF SPECIMENS 30-1 AND 30-2 BEFORE TEST

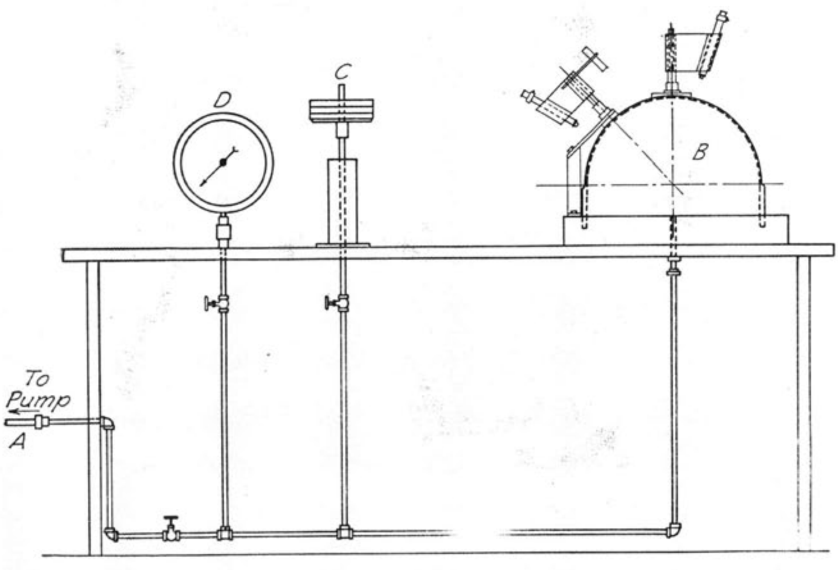

FIG. 5. APPARATUS FOR TESTING HEMISPHERES

rotated about a central axis, and so that the line of sight was normal to the shell. There were targets on the shell, one at each end of each gage line, located so that the microscope focused on the targets when in diametrically opposite positions. The sum of the increments of the microscope readings on the two targets indicated the strain on a

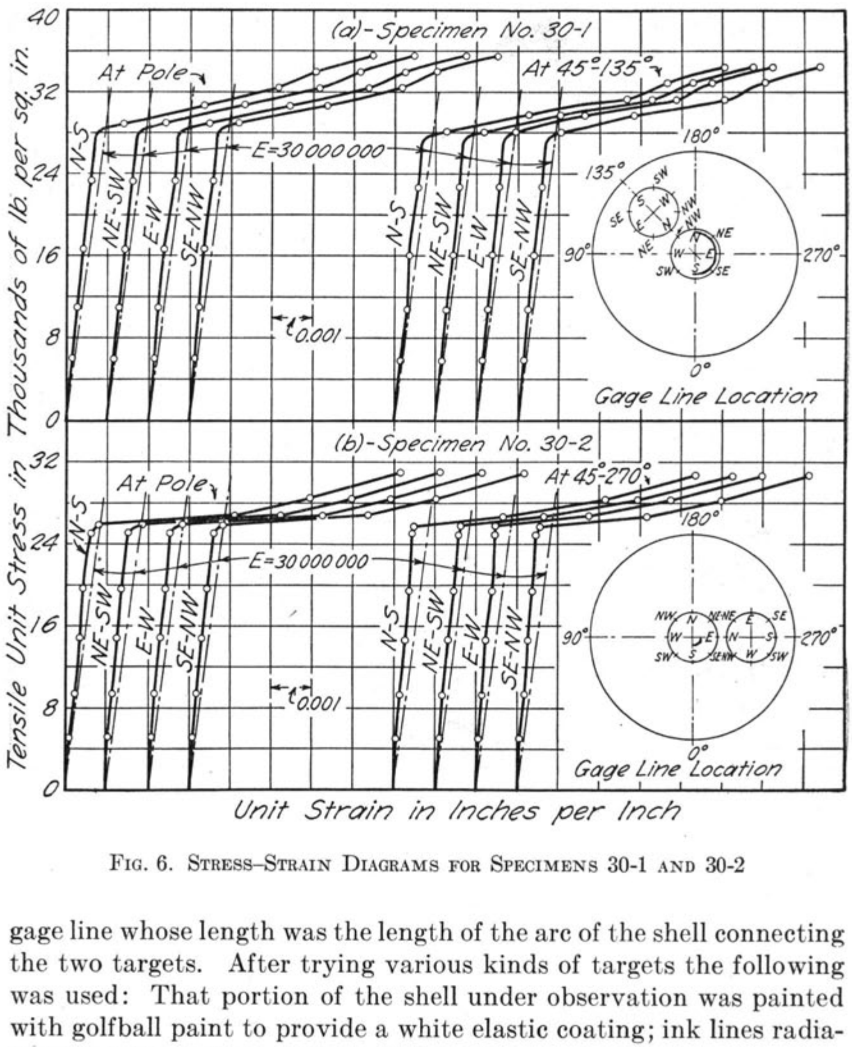

FIG. 6. STRESS–STRAIN DIAGRAMS FOR SPECIMENS 30-1 AND 30-2

gage line whose length was the length of the arc of the shell connecting the two targets. After trying various kinds of targets the following was used: That portion of the shell under observation was painted with golfball paint to provide a white elastic coating; ink lines radiating from the axis of rotation of the microscope were then drawn on this painted surface, terminating in the arc described by the line of sight of the microscope. The ends of these lines proved to be highly satisfactory as targets.

Strain readings were taken with each strain gage on four gage lines spaced 45 degrees apart, as indicated in Fig. 6. The stress in the steel was computed from the thickness of the shell and the hydrostatic pressure. The resulting stress–strain diagrams are given in Fig. 6. The similarity of the various diagrams apparently would

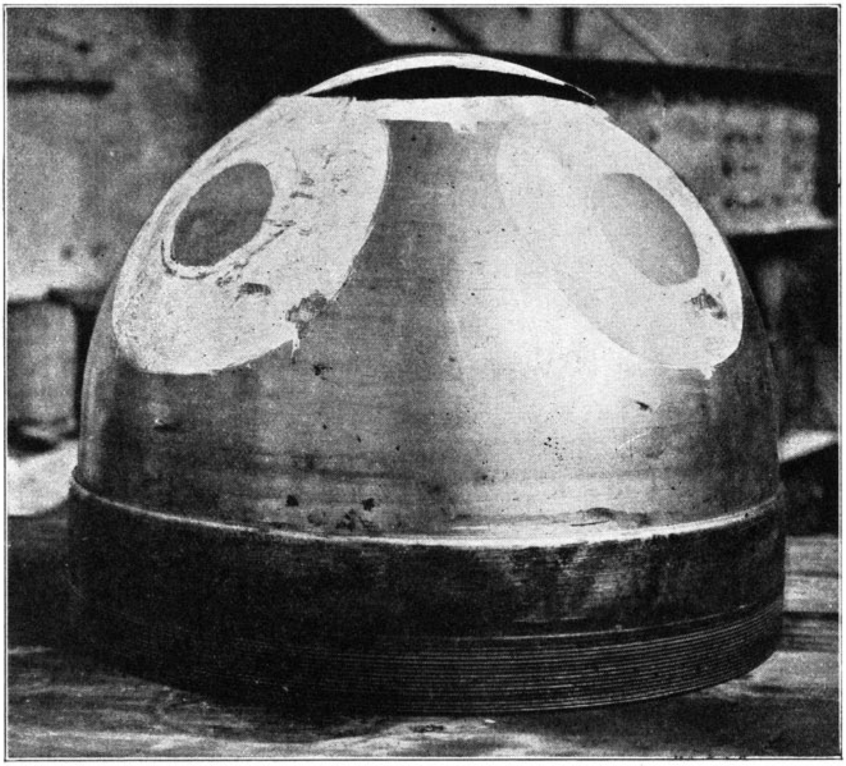

FIG. 7. SPECIMEN 30-1 AFTER FAILURE

indicate that the experimentation had been quite successful. The fact that the slope of the stress–strain diagrams exceeds 30 000 000 lb. per sq. in. is due to the Poisson's ratio effect.

The load upon the specimen was increased in successive increments, as indicated by the points on the stress–strain diagram, until the limit of the strain gages had been reached. The instruments were then removed and the hydrostatic pressure was increased until failure occurred. The appearance of specimen 30-1 after failure is shown in Fig. 7. Specimen 30-2 developed two points of failure widely removed from each other, as indicated in the drawing of Fig. 8b. One, a crack about 1 inch long which developed near the top of the specimen, allowed water to squirt out in considerable volume; the other, which was not detected until after the shell had been removed from the base, showed that necking had begun at a point near the junction with the cylindrical portion of the shell, as indicated by the short radial

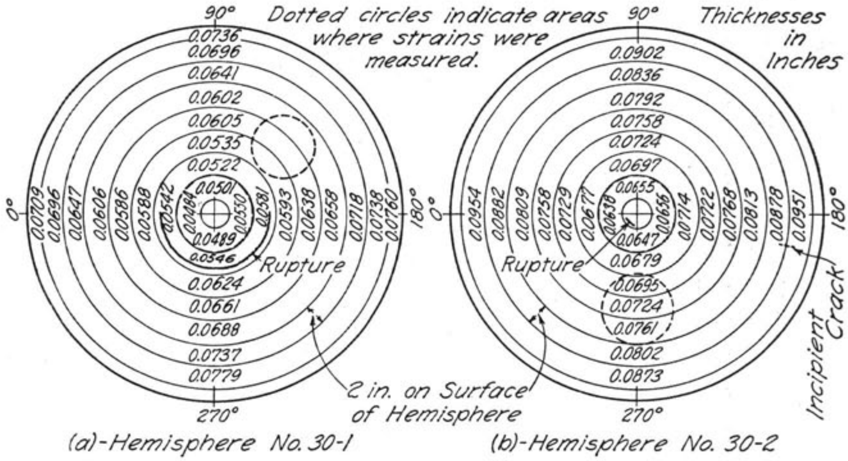

FIG. 8. THICKNESS OF SPECIMENS 30-1 AND 30-2 AFTER TEST

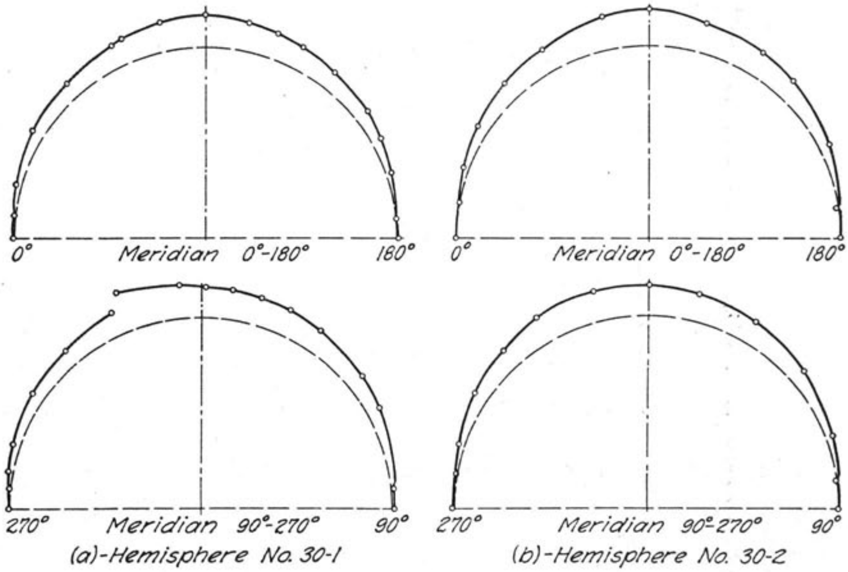

FIG. 9. PROFILES OF SPECIMENS 30-1 AND 30-2 AFTER FAILURE

line in the figure. No leak occurred at this point of incipient failure but the leakage at the other point was so great that the rupture could not be enlarged enough to show in a photograph. The thickness of the shells after failure is given in Fig. 8.

TABLE 2

COMPARISON OF THE VALUES OF THE TENSILE STRENGTH OF STEEL DEVELOPED BY HEMISPHERICAL SHELLS AND BY TENSION CONTROL SPECIMENS

Criterion of Failure	Hemisphere		Control Specimen		$R = A/B$
	Specimen No.	Stress lb. per sq. in. A	Specimen No.	Stress lb. per sq. in. B	
Stress at which unit strain equals 0.002 in. per in.	30-1	29 600 29 600 29 600 29 600	1 2 3 4	26 300 26 200 27 000 26 500	1.12
	30-2	26 200 26 100 26 200 26 100	1 2 3 4	23 600 25 000 25 400 23 000	1.08
Failure	30-1	56 100		52 700	1.10
	30-2	52 600		52 575	1.00

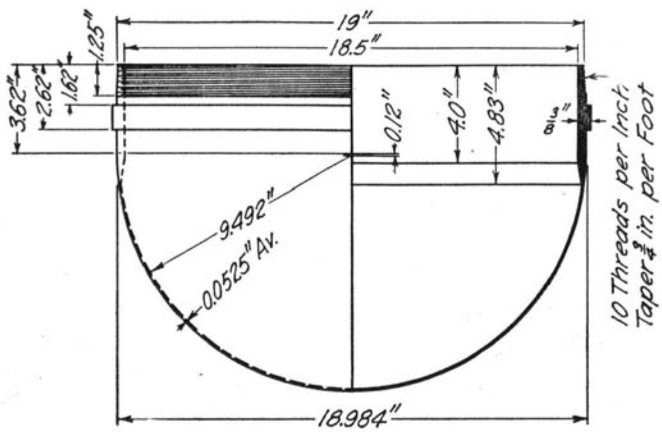

FIG. 10. FINISHED SPECIMENS 3 AND 4

The profiles of the specimens were determined after the tests had been completed, and the profiles before and after testing are compared in Fig. 9.

As indicated in Fig. 2, the cylindrical portion of the shell was much thicker than the spherical portion. Consequently the material in the cylindrical portion was not stressed to the elastic limit during the test. Control specimens were cut from this low-stressed material from which the yield point and ultimate strength of the material under uni-lateral loading was determined. The yield point and ultimate strength of the shells and the control specimens are compared

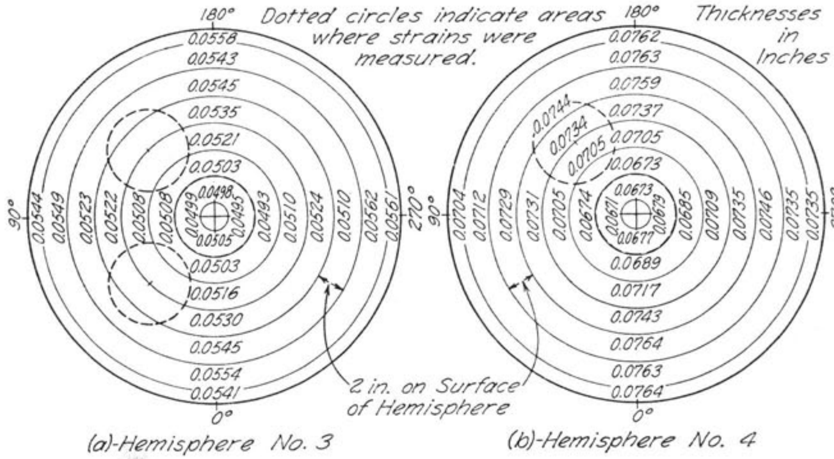

FIG. 11. THICKNESS OF SPECIMENS 3 AND 4 BEFORE TEST

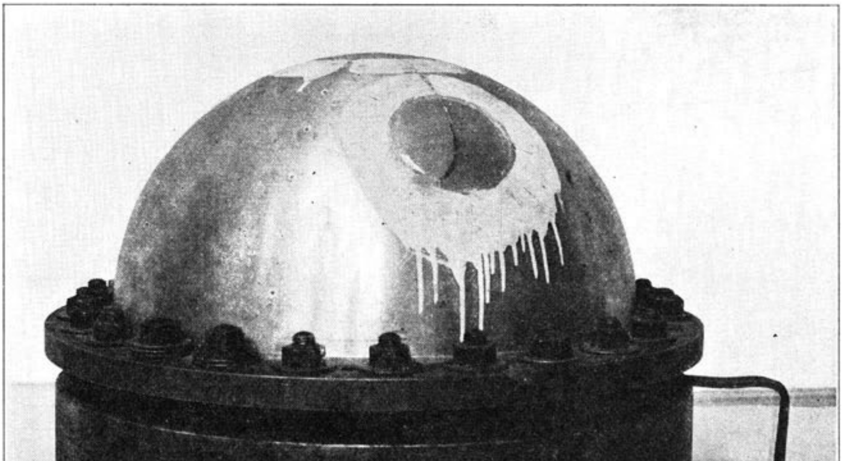

FIG. 12. STEEL RING FOR HOLDING ALUMINUM SPECIMEN IN PLANE BASE

in Table 2. These results apparently indicate that the steel of structural grade used in these tests will develop a somewhat greater yield strength in a hemispherical shell, where it is subjected to the same unit stress in all directions in a plane, than in a standard tension

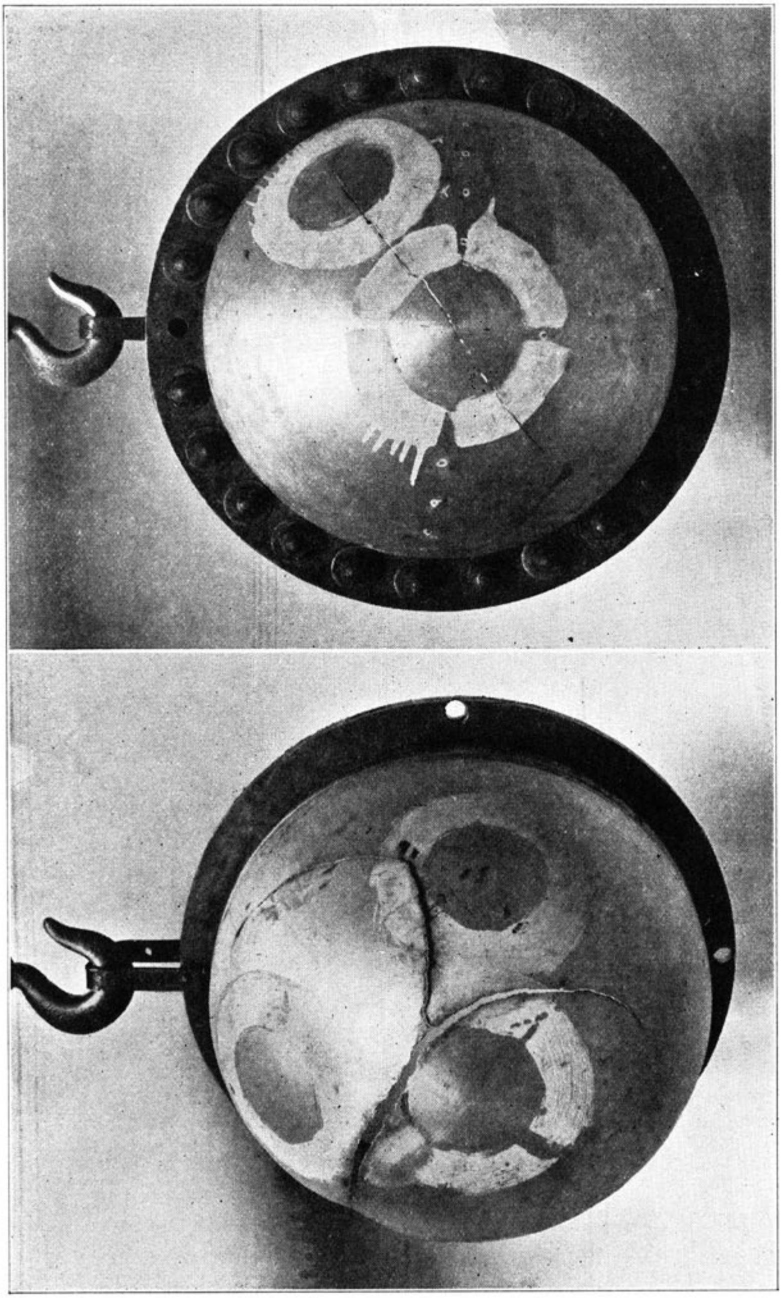

FIG. 13. SPECIMENS 3 AND 4 AFTER FAILURE

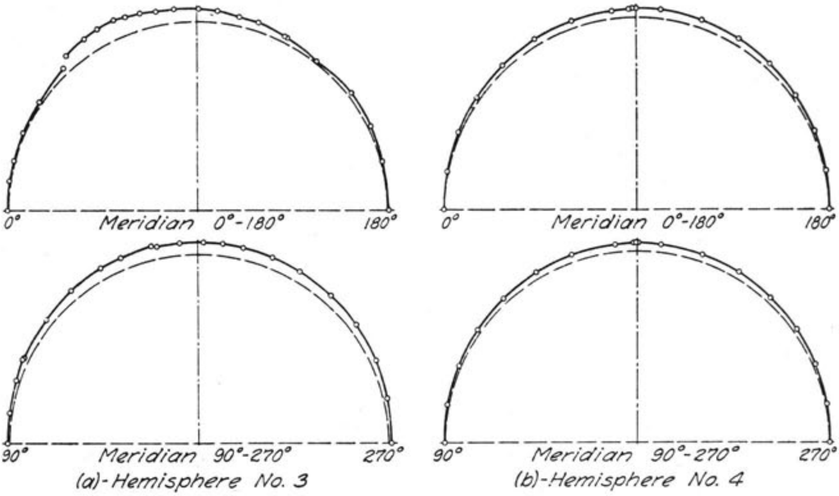

FIG. 14. PROFILES OF SPECIMENS 3 AND 4 AFTER FAILURE

specimen, where it is subjected to a stress in only one direction; and the ultimate strength is as great in a hemispherical shell as in a standard tension control specimen, or possibly a little greater.

4. *Tests of Aluminum Shells.*—Two specimens were made of an aluminum alloy designated by the trade name 51S, one of the stronger alloys of aluminum. The shells were spun to approximately the same dimensions as the steel shells, shown in Fig. 1, and machined to the nominal dimensions shown in Fig. 10. The exact thickness of the shells at various points is given in Fig. 11.

The aluminum specimens were loaded in the same manner as the steel ones, but the threads holding the specimen in the base would not resist the load to which they were subjected. Four attempts were made on the first of the aluminum shells, specimen No. 3, before it was tested to failure. The second aluminum shell, specimen No. 4, was held in the base by means of a steel ring bearing upon a shoulder on the cylindrical portion of the shell, as shown in Fig. 12.

The appearance of specimens 3 and 4 after failure is shown in Fig. 13, the change in the profiles in Fig. 14, and the thickness of the shells after failure in Fig. 15.

Tension control specimens were cut from the cylindrical portion of the shells after the tests had been completed. The stress–strain diagrams are given in Fig. 16 for the control specimens, and in Fig. 17 for the shells.

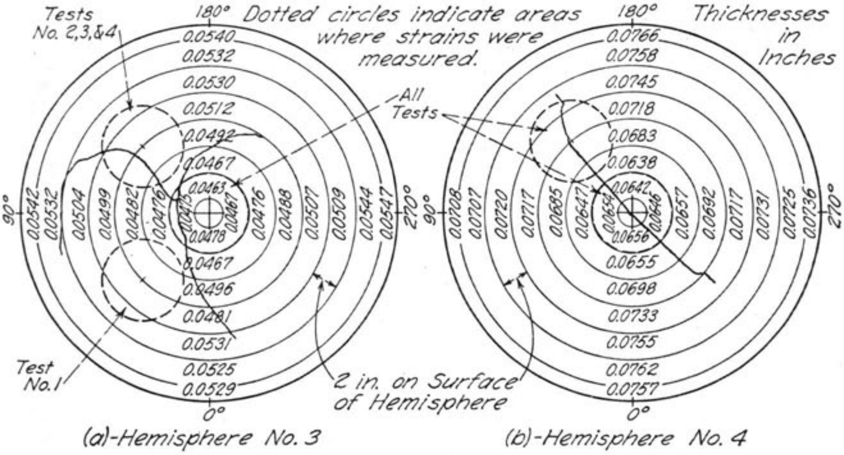

FIG. 15. THICKNESS OF SPECIMENS 3 AND 4 AFTER FAILURE

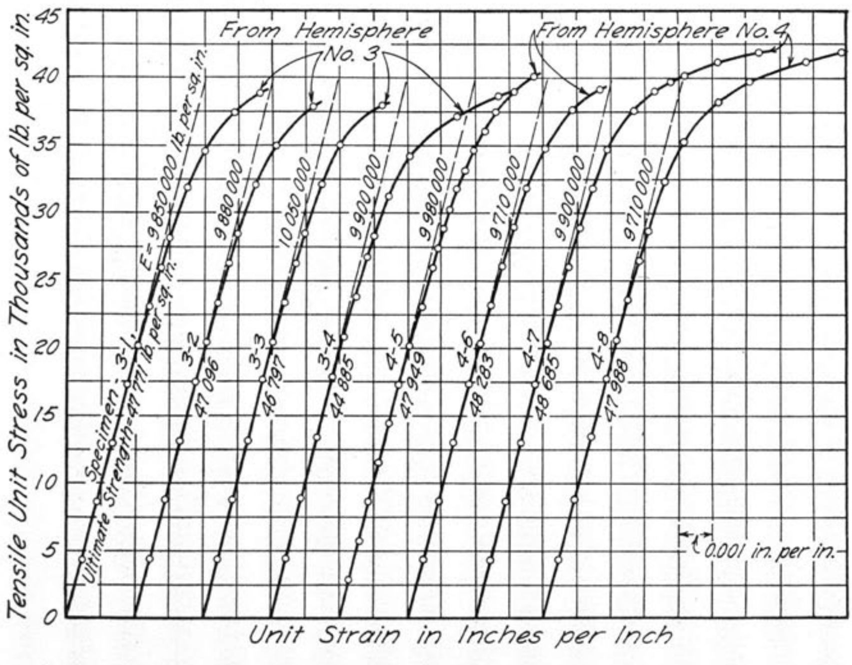

FIG. 16. STRESS–STRAIN DIAGRAMS FOR CONTROL SPECIMENS, SPECIMENS 3 AND 4

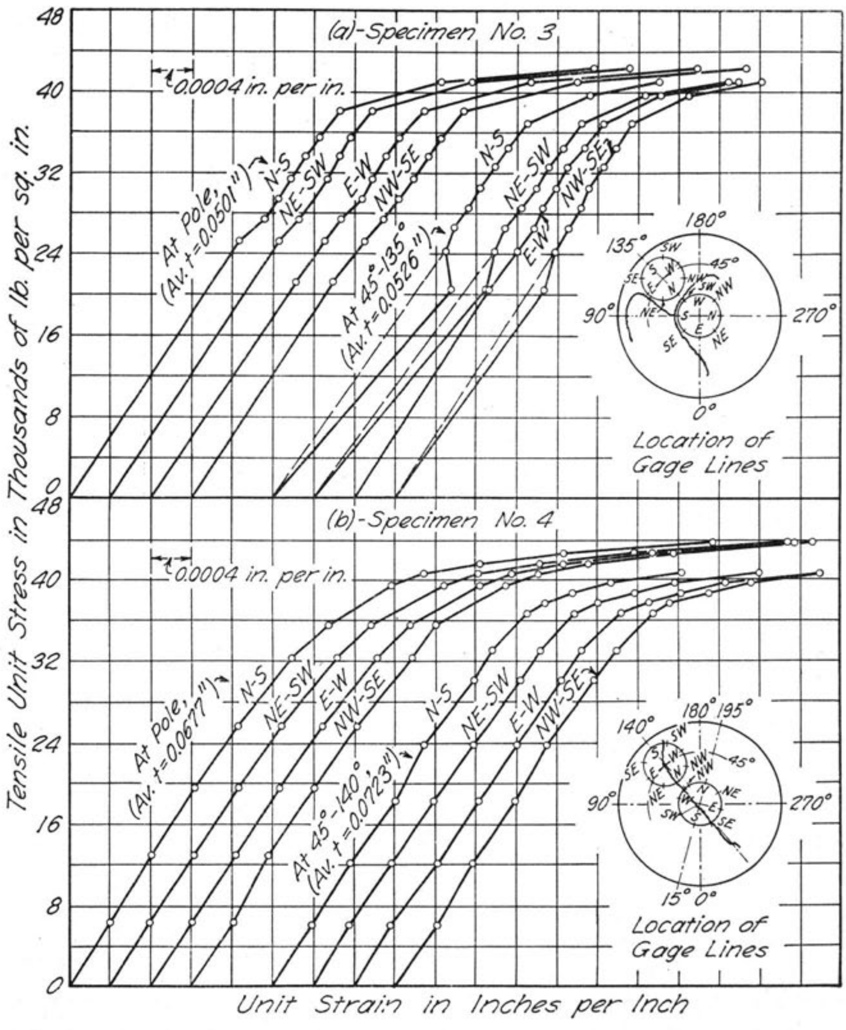

FIG. 17. STRESS–STRAIN DIAGRAMS FOR SHELLS, SPECIMENS 3 AND 4

The stress producing a unit strain of 0.004 and the ultimate strength, for the shells and the control specimens, are compared in Table 3. It would appear from the values in this table that the aluminum alloy used in these tests developed a greater yield strength in a hemispherical shell, where it is subjected to the same unit stress in all directions in a plane, than in a standard tension specimen, where it is subjected to a stress in only one direction; and that the

TABLE 3
COMPARISON OF THE VALUES OF THE TENSILE STRENGTH OF ALUMINUM ALLOY DEVELOPED BY HEMISPHERICAL SHELLS AND BY TENSION CONTROL SPECIMENS

Criterion of Failure	Hemisphere		Control Specimen		$R = A/B$
	Specimen No.	Stress lb. per sq. in. A	Specimen No.	Stress lb. per sq. in. B	
Stress at which strain equals 0.004 in. per in.	3	41 200 41 400 41 200 41 200	1 2 3 4	34 500 34 400 35 000 34 000	1.20
	4	41 500 41 100 41 300 41 600	1 2 3 4	35 000 34 500 35 400 34 600	1.19
Failure	3	50 180	....	46 640	1.10
	4	49 350	....	48 200	1.02

ultimate strength is as great in a hemispherical shell as in a standard tension control specimen, or possibly a little greater.

5. *Comparison of Measured and Computed Strength.*—The unit strength of four hemispherical shells has been determined from the tests described in Sections 3 and 4, and the values of R, the ratio of the strength developed by a material in the shell to the strength of the control specimens, is given in Tables 2 and 3. Although the tests were planned to get information governing the design of a particular structure, a spherical shell subjected to an internal pressure, and were not planned to check the various theories of failure, nevertheless it is of interest to compare the values of R determined from these tests with the values of R computed from the various theories of failure.

In the following discussion the strength of the material will be taken as the yield point or the yield strength. Under the condition of combined stress which exists in the hemispherical shells an important question is the mutual effect of stresses at right-angles to each other. Several theories of failure have been proposed in an attempt to explain the failure of materials subjected to combined stresses.*† For ductile metals recent tests† show that the shear-energy theory is in close agreement with test results. Other theories often used for design are the shear theory, the stress theory, the strain theory, and the strain-energy theory. The results of the tests that have been

*S. Timoshenko—"Strength of Materials," Vol. 2, Van Nostrand.
†J. Marin—"Failure Theories of Materials Subjected to Combined Stresses." Trans., A.S.C.E., Vol. 101, p. 1162.

described will be compared with the results obtained on the bases of these five theories.

The shear theory is based on the assumption that failure occurs when the shear stress under biaxial loading equals the shear stress at failure in simple tension. According to this theory, if the two principal stresses are equal, the strength under biaxial loading is the same as the strength in simple tension; that is, $R = 1$.

The stress theory is based on the assumption that failure occurs when the tensile stress (compression is considered a negative tension) under biaxial loading equals the tensile stress at failure in simple tension. According to this theory also, if the two principal stresses are equal, as for the hemispherical shells, the strength under biaxial loading is the same as the strength in simple tension, and $R = 1$.

The strain theory is based on the assumption that failure occurs when the strain in the direction of the maximum principal stress reaches the value of the strain at failure in simple tension. Under this theory, if the two principal stresses are equal, the strength under biaxial loading equals the strength in simple tension divided by one minus Poisson's ratio, which has values of approximately 0.28 for steel and 0.33 for aluminum. Using these values for Poisson's ratio, the values of R would be 1.39 and 1.49 for steel and aluminum, respectively.

The shear-energy theory is based on the assumption that failure occurs when the energy due to shear distortion under combined stress equals the energy of shear distortion for simple tension. Under this theory, if the two principal stresses are equal, the strength under biaxial loading equals the strength in simple tension, and $R = 1$ for both steel and aluminum.

The strain-energy theory is based on the assumption that failure occurs when the total internal strain energy absorbed equals the total strain energy absorbed at failure in simple tension. Under this theory, if the two principal stresses are equal, the strength under biaxial loading equals the strength in simple tension divided by $\sqrt{2\ (1 - \text{Poisson's ratio})}$. The values of R would be 0.83 and 0.86 for steel and aluminum, respectively.

Values of the stress ratio R determined by the tests, for both the steel and aluminum specimens, are given in the last column of Tables 2 and 3. These values are compared with the values obtained by the various theories in Table 4.

As stated previously, these tests were not planned to check the various theories of failure and their value for this purpose is quite

TABLE 4
COMPARISON OF MEASURED AND COMPUTED VALUES OF STRENGTH OF HEMISPHERICAL SHELLS

Method of Determination	Values of R		Percentage by Which Strength Computed by Various Theories Exceeds Measured Strength	
	Steel	Aluminum	Steel	Aluminum
Shear Theory	1.0	1.0	− 9.1	−16.3
Stress Theory	1.0	1.0	− 9.1	−16.3
Shear-Energy Theory	1.0	1.0	− 9.1	−16.3
Strain Theory	1.39	1.49	+26.4	+24.1
Strain-Energy Theory	0.83	0.86	−24.5	−28.3
Tests	1.10	1.20	....	....

$R = \dfrac{\text{Unit strength in shell}}{\text{Unit strength in simple tension}}$

Poisson's ratio is taken as 0.28 for steel and 0.33 for aluminum.

limited. A study of Table 4, however, reveals the following facts relative to the results from this rather small number of tests.

The shear theory, the stress theory, and the shear-energy theory, which give the same results for equal stresses in all directions in a plane, give a strength for the hemispherical shells that is a little less than the strength found by the tests, the difference being 9.1 per cent for the steel shells, and 16.3 per cent for the aluminum shells. The strain theory gives values for the strength of the shell that are too large, being 26 per cent too large for the steel, and 24 per cent too large for the aluminum shells. The strain-energy theory gives values for the strength of the shells that are only about three-fourths as great as the values given by the tests.

III. CONCLUSIONS

6. *Conclusions.*—If a thin spherical shell of ductile material subjected to an internal pressure is designed on the basis that the allowable unit stress in the shell equals the allowable unit stress in the same material when subjected to simple tension, the design will be safe.

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